# REPORT DOCUMENTATION PAGE

*Form Approved*
*OMB No. 0704-0188*



1. AGENCY USE ONLY *(Leave Blank)*

2. REPORT DATE
23-Jan-02

3. REPORT TYPE AND DATES COVERED
Final; 01-Apr-99 through 31-Mar-01

4. TITLE AND SUBTITLE
Development and Validation of a Stochastic Model for the Hydrodynamic Forcing Function from Submarine Propulsors and Appendages

5. FUNDING NUMBERS
G; N00014-99-1-0588

6. AUTHORS
Leonard Joel Peltier

7. PERFORMING ORGANIZATION NAME(S) AND ADDRESS(ES)
Applied Research Laboratory
The Pennsylvania State University
PO Box 30
State College, PA 16804

8. PERFORMING ORGANIZATION REPORT NUMBER

9. SPONSORING / MONITORING AGENCY NAME(S) AND ADDRESS(ES)
Office of Naval Research, ONR 333
Ballston Centre Tower One
800 North Quincy Street
Arlingtoin, VA 22217-5660

10. SPONSORING / MONITORING AGENCY REPORT NUMBER

11. SUPPLEMENTARY NOTES

12a. DISTRIBUTION / AVAILABILITY STATEMENT
Approved for public release

12b. DISTRIBUTION CODE

13. ABSTRACT *(Maximum 200 words)*

The long-term objective of this research was to develop a stochastic model for the two-point surface-pressure correlation and the related surface-pressure spectrum that uses RANS data as input. The model would have given a complete hydrodynamic forcing function for structural acoustics applications. The use of Reynolds-Averaged Navier-Stockes data as input to the model allowed the model to respone to local flow and configuration complications through the RANS modeling. Contemporary experience with RANS shows clearly that it can predict mean-field statistics for Navy applications with reasonable fidelity. The method is first-principles-based and is expected to be valid for evaluation of new designs, ones with no already accumulated experimental database.

14. SUBJECT TERMS
Stochastic Model for the Hydrodynamic Forcing Function
Submarine Propulsors and Appendages
Reynolds-Averaged Navier-Stokes (RANS)

15. NUMBER OF PAGES
16

16. PRICE CODE

17. SECURITY CLASSIFICATION OF REPORT
unclassified

18. SECURITY CLASSIFICATION OF THIS PAGE
unclassified

19. SECURITY CLASSIFICATION OF ABSTRACT
unclassified

20. LIMITATION OF ABSTRACT
SAR

NSN 7540-01-280-5500

Standard Form 298 (Rev. 2-89)
Prescribed by ANSI Std. Z39-1
298-102

20020131 070

**Final Report:**
**Development and Validation of a Stochastic Model for the Hydrodynamic Forcing Function from Submarine Propulsors and Appendages**

Leonard Joel Peltier, Research Associate
Computational Mechanics, Applied Research Laboratory
The Pennsylvania State University, PO Box 30
State College, PA 16804, USA
TEL: (814) 863-3014, FAX: (814) 865-3287
EMAIL: ljp@wt.arl.psu.edu

*ONR Grant #N00014-99-1-0588*
*L. Patrick Purtell, Program Officer*

**Long-Term Research Objective**
The long-term objective of this research was to develop a stochastic model for the two-point surface-pressure correlation and the related surface-pressure spectrum that uses RANS data as input. The model would have given a complete hydrodynamic forcing function for structural acoustics applications. The use of Reynolds-Averaged Navier-Stokes data as input to the model allowed the model to respond to local flow and configuration complications through the RANS modeling. Contemporary experience with RANS shows clearly that it can predict mean-field statistics for Navy applications with reasonable fidelity. The method is first-principles-based and is exptected to be valid for evaluation of new designs, ones with no already accumulated experimental database.

The work was to build on an existing model for the surface-pressure autospectrum developed at ARL under internal funds.

**Work Achieved**
Only preliminary funding for the project was received before the grant was rescinded, due to a similar effort also in progress under ONR funding.

Under the funding received, the formulation for the two-point autospectrum was developed and the preliminary models for the correlation functions were constructed, see attached viewgraph report given to P. Purtell, contract manager. Effort during the first phase of funding focused on understanding anisotropy of the near wall flow field, especially relating to the buffer layer and early logarithmic layer of the turbulent boundary-layer flow. Near-wall anisotropy was expected to be important for modeling the surface-pressure covariance, since the correlation lengths between these layers differ significantly.

The initial increment of funding did not allow for a completed model to be developed. The understanding and modeling improvements gained from the increment of funding, however, were implemented in the ARL stochastic surface-pressure autospectrum model, improving it significantly. A plot showing the final surface-pressure autospectrum model (incorporating the results of this grant) comparing it to data for a range of Reynolds numbers demonstrates that the model reproduces data reasonably well for all wavenumbers. Significant differences are within the envelope of

experimental uncertainty.

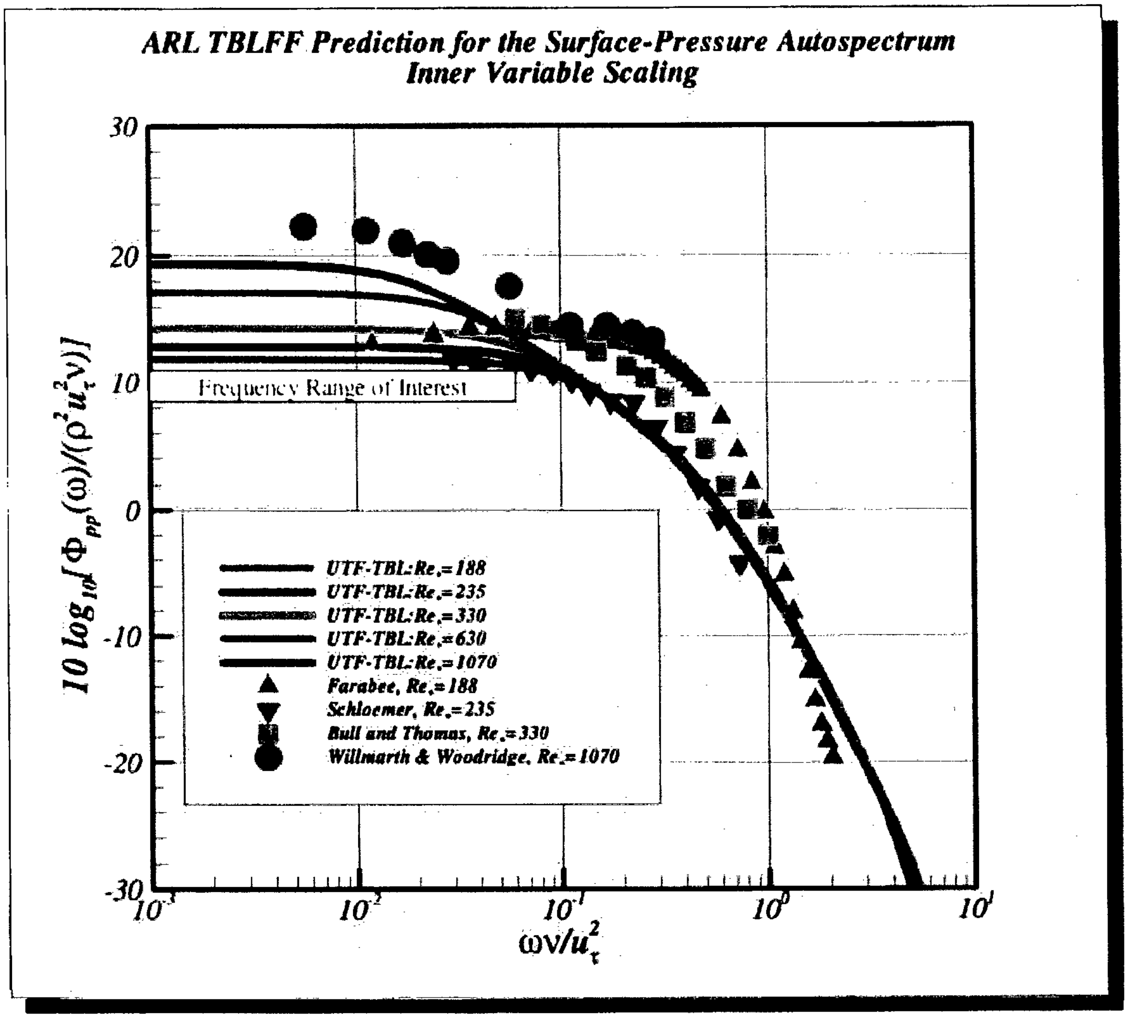


Figure 1: Stochastic model for the surface-pressure autospectrum versus data.

# Appendix

PENNSTATE ARL

Computational Mechanics

# Stochastic Modeling of the Hydrodynamic Forcing Function Using RANS Data as Input


Presented by:

L. Joel Peltier, Research Associate

Computational Mechanics, Applied Research Laboratory

The Pennsylvania State University

By taking the divergence of the Navier-Stokes Equation

$$\underbrace{u_{i,t}}_{\textit{TimeChange}} + \underbrace{(u_i u_j)_{,j}}_{\textit{Advection}} = \underbrace{-p_{,i}}_{\textit{Presure}} + \underbrace{\nu\, u_{i,jj}}_{\textit{Viscous Diffusion}}$$

and accounting for Continuity

$$u_{i,i} = 0 \quad ,$$

we can write the pressure Poisson equation

$$p_{,ii} = -(u_i u_j)_{,ij} = \textit{Purely Inertial Terms}.$$

A notation is adopted such that a full field (bold) is decomposed into an ensemble-mean contribution (upper case) and a fluctuation (lower case). For example,

$$\underbrace{\boldsymbol{u}_i}_{\substack{\textit{Full}\\ \textit{Velocity}}} = \underbrace{U_i}_{\substack{\textit{Mean}\\ \textit{Velocity}}} + \underbrace{u_i}_{\substack{\textit{Fluctuating}\\ \textit{Velocity}}} \quad .$$

Substitution and simplification yields, the Poisson equation governing pressure fluctuations

$$p_{,ii} = -\Big( \underbrace{U_i u_j + u_i U_j}_{\substack{\textit{Turbulence-Mean-Shear}\\ \textit{Interaction}}} + \underbrace{u_i u_j - \overline{u_i u_j}}_{\substack{\textit{Turbulence-Turbulence}\\ \textit{Interaction}}} \Big)_{,ij} \quad .$$

This equation can be integrated using the Green's Function technique. For a suitable Green's Function, G,

$$p = \int_{x} -(U_i u_j + u_i U_j + u_i u_j - \overline{u_i u_j})_{,ij}\ G\ dx \quad ,$$

The flat-plate Green's Function is

$$G(x, x_s) = -\frac{1}{2\pi} \frac{1}{\| x - x_s \|} \quad ,$$

which we will assume is generally applicable to faceted surfaces where the facet lengths are greater than the local boundary-layer thicknesses (i.e. we are focused on discrete surfaces like the representations in computational grids).

Other Green's Function choices are available, if needed (see M. Howe, Boston University).

The surface pressure at $\mathbf{x}_s$ is simply

$$p_s\,(x_s,\,t) = \int_x -(U_i u_j + u_i U_j + u_i u_j - \overline{u_i u_j})_{,ij}\; G(x,\,x_s)\; dx \quad .$$

At a second surface point, the surface pressure is

$$p_s\,(y_s,\,\tau) = \int_y -(V_k v_l + v_k V_l + v_k v_l - \overline{v_k v_l})_{,kl}\; G(y,\,y_s)\; dy \quad .$$

Their covariance is

$$\begin{aligned}\overline{p_s(x_s,\,t)\,p_s(y_s,\,\tau)} = \int_y\!\int_x \quad & < (U_i u_j + u_i U_j + u_i u_j - \overline{u_i u_j})_{,ij} \\ & \times (V_k v_l + v_k V_l + v_k v_l - \overline{v_k v_l})_{,kl} > \quad . \\ & \times G(x,\,x_s,\,t)\; G(y,\,y_s,\,\tau) \qquad dx\,dy\end{aligned}$$

Angle brackets and overlines denote an ensemble averaging.

Useful manipulations for simplifying the integrand are

$$< (U_i u_j + u_i U_j + u_i u_j - \overline{u_i u_j})_{,ij} \, (V_k v_l + v_k V_l + v_k v_l - \overline{v_k v_l})_{,kl} >$$

$$= \; < (U_i u_j + u_i U_j + u_i u_j - \overline{u_i u_j})(V_k v_l + v_k V_l + v_k v_l - \overline{v_k v_l}) >_{,ijkl}$$

and

$$< (U_i u_j + u_i U_j + u_i u_j - \overline{u_i u_j})(V_k v_l + v_k V_l + v_k v_l - \overline{v_k v_l}) >$$

$$\begin{aligned} = \; & U_i V_k \overline{u_j v_l} + U_i V_l \overline{u_j v_k} + U_i \overline{u_j v_k v_l} - U_i \overline{u_j} \, \overline{v_k v_l} \\ & + U_j V_k \overline{u_i v_l} + U_j V_l \overline{u_i v_k} + U_j \overline{u_i v_k v_l} - U_j \overline{u_i} \, \overline{v_k v_l} \\ & + V_k \overline{u_i u_j v_l} + V_l \overline{u_i u_j v_k} + \overline{u_i u_j v_k v_l} - \overline{u_i u_j} \, \overline{v_k v_l} \\ & - V_k \overline{u_i u_j} \, \overline{v_l} - V_l \overline{u_i u_j} \, \overline{v_k} - \overline{u_i u_j} \, \overline{v_k v_l} + \overline{u_i u_j} \, \overline{v_k v_l} \end{aligned}$$

which when coupled with the modeling assumptions

- if the velocity probability distribution is nearly symmetric triple products may be neglected (see Reginald Hill of NOAA for limitations)

- if the velocity components are normally distributed, the fourth-order moment can be written as products of the second-order moments (a property of the normal distribution

yield the

$$\overline{p_s(x,t)\,p_s(y,\tau)} = \int_y \int_x \left( \begin{array}{l} \quad U_i V_k \overline{u_j v_l} + U_i V_l \overline{u_j v_k} \\ + \; U_j V_k \overline{u_i v_l} + U_j V_l \overline{u_i v_k} \\ + \; \overline{u_j v_k}\,\overline{u_i v_l} + \overline{u_j v_l}\,\overline{u_i v_k} \end{array} \right)_{,ijkl} G(x, x_s, t)\, G(y, y_s, \tau)\, dx\, dy$$

To complete the model, we must

- choose an analytic form for the velocity correlation function which satisfies the physics we hope to capture,
    - Tom Gatski's Algebraic Stress Model (ASM) is being used to introduce realistic anisotropy into the model.
    - An appropriate prescription for the correlation lengths is being sought.
- choose a Green's Function form appropriate for the intended geometry,
- then integrate the expression for all separations **y-x**.

Development of a Stochastic Model for the Hydrodynamic Forcing Function using RANS Data as Input

Let $\tilde{u}_i = U_i + u_i$

Full- Mean- fluctuating- velocity

$$\tilde{u}_{i,t} + (\tilde{u}_i \tilde{u}_j)_{,j} = -\tilde{p}_{,i} + \nu \tilde{u}_{i,jj}$$

Navier-Stokes Equation

From the divergence of the N-S budget, we have the pressure Poisson equation

$$\tilde{p}_{,ii} = -(\tilde{u}_i \tilde{u}_j)_{,ij}$$

Separate $\tilde{p}_{,ii}$ into its mean and fluctuating parts

The fluctuating-pressure Poisson equation is

$$p_{,ii} = -(\underbrace{\overline{U}_i u_j + u_i \overline{U}_j}_{\text{Turbulence-Mean Interactions}} + \underbrace{u_i u_j - \overline{u_i u_j}}_{\text{Turbulence-Turbulence Interactions}})_{,ij}$$

An expression for pressure is obtained by integration using the Green's Function approach

$$p = \int_V -(\overline{U}_i u_j + u_i \overline{U}_j + u_i u_j - \overline{u_i u_j})_{,ij}\, G \, dV$$

An appropriate Green's Function, $G$, must be used. For simplicity, we will use a flat-plate $G$

$$G(\tilde{x}, \tilde{x}_s) = -\frac{1}{2\pi} \frac{1}{\| \tilde{x}_s - \tilde{x} \|}$$

to estimate the surface pressure

$$p_s = \int_V -(\overline{U}_i u_j + u_i \overline{U}_j + u_i u_j - \overline{u_i u_j})_{,ij}\, G(\tilde{x}, \tilde{x}_s)\, dV$$

Use of other geometry-specific Green's Function choices will be explored. M. Howe will be consulted.

Expressions for surface-pressures at points $\tilde{x}_s$ and $\tilde{y}_s$ are

$$p_s(\tilde{x}_s, t) = \int_{\tilde{x}} -(\bar{U}_i u_j + u_i \bar{U}_j + u_i u_j - \overline{u_i u_j})_{,ij}\, G(\tilde{x}, \tilde{x}_s, t)\, d\tilde{x}$$

$$p_s(\tilde{y}_s, \tau) = \int_{\tilde{y}} -(\bar{V}_k v_l + v_k \bar{V}_l + v_k v_l - \overline{v_k v_l})_{,kl}\, G(\tilde{y}, \tilde{y}_s, \tau)\, d\tilde{y}$$

The surface-pressure cross-correlation function is formed from their ensemble-meaned product.

$$\langle p_s(\tilde{x}_s, t)\, p_s(\tilde{y}_s, \tau) \rangle =$$

$$\int_{\tilde{y}} \int_{\tilde{x}} \langle (\bar{U}_i u_j + u_i \bar{U}_j + u_i u_j - \overline{u_i u_j})_{,ij}$$

$$\times (\bar{V}_k v_l + v_k \bar{V}_l + v_k v_l - \overline{v_k v_l})_{,kl} \rangle$$

$$\times G(\tilde{x}, \tilde{x}_s, t)\, G(\tilde{y}, \tilde{y}_s, \tau) \quad d\tilde{x}\, d\tilde{y}$$

which expands to

$$\langle p_s(\tilde{x}_i, t)\, p_s(\tilde{y}_j, \tau) \rangle =$$

$$\int_{\tilde{y}} \int_{\tilde{x}} \Big( U_i V_k \overline{u_j v_l} + U_i V_l \overline{u_j v_k} + U_i \overset{?}{\overline{u_j v_k v_l}} - U_i \overline{v_k v_l}\, \cancel{\bar{u}_j}^{\,0}$$

$$+ U_j V_k \overline{u_i v_l} + U_j V_l \overline{u_i v_k} + U_j \overset{?}{\overline{u_i v_k v_l}} - U_j \overline{v_k v_l}\, \cancel{\bar{u}_i}^{\,0}$$

$$+ V_k \overset{?}{\overline{u_i u_j v_l}} + V_l \overset{?}{\overline{u_i u_j v_k}} + \overline{u_i u_j v_k v_l} - \overline{u_i u_j}\; \overline{v_k v_l}$$

$$- V_k \overline{u_i u_j}\, \cancel{\bar{v}_l}^{\,0} - V_l \overline{u_i u_j}\, \cancel{\bar{v}_k}^{\,0} \overbrace{- \overline{u_i u_j}\; \overline{v_k v_l} + \overline{u_i u_j}\; \overline{v_k v_l}}^{0} \Big)_{,ijkl}$$

$$\times\ G(\tilde{x}, \tilde{x}_s, t)\, G(\tilde{y}, \tilde{y}_s, \tau)\ d\tilde{x}\, d\tilde{y}$$

Triple products may be neglected if the probability distribution of velocity is „nearly" symmetric.

Perhaps early work by Launder can shed some light on this assumption

Assuming that the velocity components are normally distributed, the fourth-order moment can be reduced to products of the second-order moments.

$$
\langle p_s(\tilde{x}_s, t)\, p_s(\tilde{y}_s, \tau) \rangle =
$$

$$
\int_{\tilde{y}} \int_{\tilde{x}} \Big( U_i V_k\, \overline{u_j v_l} + U_i V_l\, \overline{u_j v_k}
$$

$$
+ U_j V_k\, \overline{u_i v_l} + U_j V_l\, \overline{u_i v_k}
$$

$$
+ \overline{u_i v_k}\; \overline{u_j v_l} + \overline{u_i v_l}\; \overline{u_j v_k} \Big)_{,ijkl}
$$

$$
\times\, G(\tilde{x}, \tilde{x}_s, t)\, G(\tilde{y}, \tilde{y}_s, \tau) \quad d\tilde{x}\, d\tilde{y}
$$

A model for the velocity correlation function must be assumed; and the expression must be integrated for all separations $\tilde{y} - \tilde{x}$.

Early ARL work modeled the velocity correlation function with a Gaussian fit. The present work looks toward Julian Hunt and Jakob Mann for further guidance.